On the multi-component NLS type models and their gauge equivalent

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The fundamental properties of the multi-component nonlinear Schrödinger (NLS) type models related to semi-simple Lie algebra $g$

\[ L(\lambda)\psi(x,t,\lambda) \equiv \left( i \frac{d}{dx} + q(x,t) - \lambda J \right) \psi(x,t,\lambda) = 0, \]

\[ M(\lambda)\psi(x,t,\lambda) \equiv \left( i \frac{d}{dt} - \pi_0 \left( [q, \text{ad}_J^{-1} q_x] \right) + 2i\text{ad}_J^{-1} q_x(x,t) + 2\lambda q(x,t) - 2\lambda^2 J \right) \psi(x,t,\lambda) = 0, \]

and their gauge equivalent Heisenberg ferromagnet type equations

\[ \tilde{L}\tilde{\psi}(x,t,\lambda) \equiv \left( i \frac{d}{dx} - \lambda S(x,t) \right) \tilde{\psi}(x,t,\lambda) = 0, \]

\[ \tilde{M}\tilde{\psi}(x,t,\lambda) \equiv \left( i \frac{d}{dt} - 2i\lambda \text{ad}_{S_x}^{-1} S - 2\lambda^2 S \right) \tilde{\psi}(x,t,\lambda) = 0, \]

are analyzed. Here $J$ is a non-regular element of the corresponding Cartan subalgebra $\mathfrak{h}$ (this means that the kernel of the operator $\text{ad}_J$ is non-commutative one); $q(x,t) \in g \setminus g_J$; $\pi_0$ is the projector onto $g_J = \ker(\text{ad}_J)$; $\lambda \in \mathbb{C}$ is a spectral parameter and

\[ \tilde{\psi}(x,t,\lambda) = g^{-1}(x,t)\psi(x,t,\lambda), \quad S(x,t) \equiv \text{Ad}_g \cdot J = g^{-1}(x,t)Jg(x,t), \quad g(x,t) = \psi(x,t,\lambda = 0). \]

We extend our approach in [1, 2] in order to implement additional reductions of these systems. To this end we first describe the scattering data properties of the relevant Lax operator $L$ which in turn determine the spectra of the corresponding recursion operator $\Lambda$. Using the expansions over the eigenfunctions of $\Lambda$ (so-called “squared solutions”) we are able to describe: a) the class of all integrable equations related to $L$; b) their class of local integrals of motion; c) their hierarchy of Hamiltonian structures.

The results are illustrated by specific examples of NLS type systems and their gauge equivalent related to the $so(5)$-algebra.

References
