Jacobi fields along harmonic maps,

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Abstract

A *harmonic map* is a mapping between Riemannian manifolds which extremizes energy; infinitesimal deformations of harmonic maps are called *Jacobi fields*. We describe the space of harmonic 2-spheres in CP^2 as a smooth submanifold of the space of all C^k maps ($k \geq 2$). There remains the question of whether all Jacobi fields along such harmonic maps are *integrable*, i.e. do they arise from variations through harmonic maps. We answer this affirmatively for harmonic 2-spheres in the complex projective plane but negatively for harmonic 2-spheres in the 4-sphere. The affirmative answer has a bearing on the behaviour of weakly harmonic E-minimizing maps from a 3-manifold to CP^2 near a singularity and on the structure of the singular set of such maps from any manifold to CP^2.