On the multi-component NLS type models and their gauge equivalent

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The fundamental properties of the multi-component nonlinear Schrödinger (NLS) type models related to semi-simple Lie algebra $\mathfrak{g}$

$$\begin{aligned} L(\lambda)\psi(x,t,\lambda) &\equiv \left(i\frac{d}{dx} + q(x,t) - \lambda J \right)\psi(x,t,\lambda) = 0, \\ M(\lambda)\psi(x,t,\lambda) &\equiv \left(i\frac{d}{dt} - \pi_0\left([q, \mathrm{ad}_J^{-1} q_x]\right) + 2i\,\mathrm{ad}_J^{-1} q_x(x,t) + 2\lambda q(x,t) - 2\lambda^2 J \right)\psi(x,t,\lambda) = 0, \end{aligned}$$

and their gauge equivalent Heisenberg ferromagnet type equations

$$\begin{aligned} \tilde{L}\tilde{\psi}(x,t,\lambda) &\equiv \left(i\frac{d}{dx} - \lambda\mathcal{S}(x,t) \right)\tilde{\psi}(x,t,\lambda) = 0, \\ \tilde{M}\tilde{\psi}(x,t,\lambda) &\equiv \left(i\frac{d}{dt} - 2i\lambda\,\mathrm{ad}_{\mathcal{S}}^{-1}\mathcal{S}_x - 2\lambda^2\mathcal{S} \right)\tilde{\psi}(x,t,\lambda) = 0, \end{aligned}$$

are analyzed. Here J is a **non-regular** element of the corresponding Cartan subalgebra $\mathfrak{h}$ (this means that the the kernel of the operator ad_J is non-commutative one); $q(x,t) \in \mathfrak{g}\backslash\mathfrak{g}_J$; π_0 is the projector onto $\mathfrak{g}_J = \ker(\mathrm{ad}_J)$; $\lambda \in \mathbb{C}$ is a spectral parameter and

$$\tilde{\psi}(x,t,\lambda) = g^{-1}(x,t)\psi(x,t,\lambda), \quad \mathcal{S}(x,t) \equiv \mathrm{Ad}_g \cdot J = g^{-1}(x,t) J g(x,t), \quad g(x,t) = \psi(x,t,\lambda=0).$$

We extend our approach in [1, 2] in order to implement additional reductions of these systems. To this end we first describe the scattering data properties of the relevant Lax operator L which in turn determine the spectra of the corresponding recursion operator Λ. Using the expansions over the eigenfunctions of Λ (so-called "squared solutions") we are able to describe: a) the class of all integrable equations related to L; b) their class of local integrals of motion; c) their hierarchy of Hamiltonian structures.

The results are illustrated by specific examples of NLS type systems and their gauge equivalent related to the $so(5)$-algebra.

References

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