Convexity Inequalities and Applications

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The starting point is the *Favard-Berwald inequality* [2], [4], [3], which asserts that

$$\frac{1}{n+1}\sup_{x\in K} f(x) \leq \frac{1}{|K|}\int_K f(x)\,dV,$$

for every continuous concave function $f : K \to \mathbb{R}_+$ defined on an arbitrary compact convex subset $K \subset \mathbb{R}^n$, of positive volume $|K|$. Equivalently, the volume of every conoid of base K and height $f(x)$ (for every $x \in K$) does not exceed the volume of the cylindroid of base K, bounded above by the hypersurface $v = f(u)$.

The aim of our talk is to discuss the significance of this result within Choquet's theory and to prove a number of extensions and refinements of it. See [1] for full details.

References

[1] S. Barza and C. P. Niculescu, *Integral Inequalities for Concave Functions.* Preprint 2004.

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[4] J. Favard, *Sur les valeurs moyennes,* Bull. Sci. Math. (2) 57 (1933), 54-64.

[5] C. P. Niculescu and Persson L.-E., *Convex Functions. Basic Theory and Applications*, Universitaria Press, 2003.

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