

Ordinary Differential Equations: decomposing and solving equations in terms of irreducible representations of symmetry algebras

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Abstract

Viewing linear ODEs as $C^\infty(\mathbb{R})$ -modules with a derivation over $\frac{d}{dx}$ is appropriate for studying symmetry algebras of the equations. Modules, and hence equations, that possess a semisimple Lie algebra of symmetries are decomposable. In particular we shall see how equations with a \mathfrak{sl}_2 symmetry algebra decompose into modules isomorphic to symmetric products of a base rank two module corresponding to a second order equation, a model equation for \mathfrak{sl}_2 . The model equations can be integrated directly, and knowing its solutions and the decomposition solves the original equation.