Fredholm Joint Spectrum for Families of Operators

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Abstract

The most convenient versions of the Taylor joint spectrum for tuples of operators generating a nilpotent Lie algebra were constructed by Feinshtein and Dosiyev [1,2]. A remarkable feature of Feinshtein's spectrum is that it satisfies a natural polynomial spectral mapping theorem with respect to polynomials in noncommuting variables.

In this work, Fredholm joint spectrum of noncommuting operator family which generates nilpotent Lie algebra is defined and investigated some properties of it. Let $a = (a_1, ...a_n)$ be define an operators family on the Banach space X, $\lambda \in C^n$, $(a - \lambda) = (a_1 - \lambda_1, ..., a_n - \lambda_n)$ and $E(a)$ be the nilpotent Lie algebra generated by a in the algebra $L(X)$. $Kos(a, X)$ is used for the Kozsul complex generated by the $E(a)$ module X [2]. The Fredholm joint spectrum of a is defined to be the set $\sigma_F(a)$ of those $\lambda \in C^n$ for which the complex $Kos(a - \lambda, X)$ whose homology spaces are not of finite dimension. It is proved that $\sigma_F(a)$ is a compact set in C^n. Polynomial spectral mapping theorem with respect to polynomials in noncommuting variables is proved such that $P(\sigma_F(a)) \subset \sigma_F(P(a))$.

References

1. DOSIYEV, A., "Spectra of infinite parametrized Banach complexes" , J. Oper. Theory 48, No.3,585-614(2002).

2. FEINSHTEIN, A.S., "Taylor joint spectrum for families of operators generating nilpotent Lie algebras", J. Operator Theory 29,3-27 (1993).